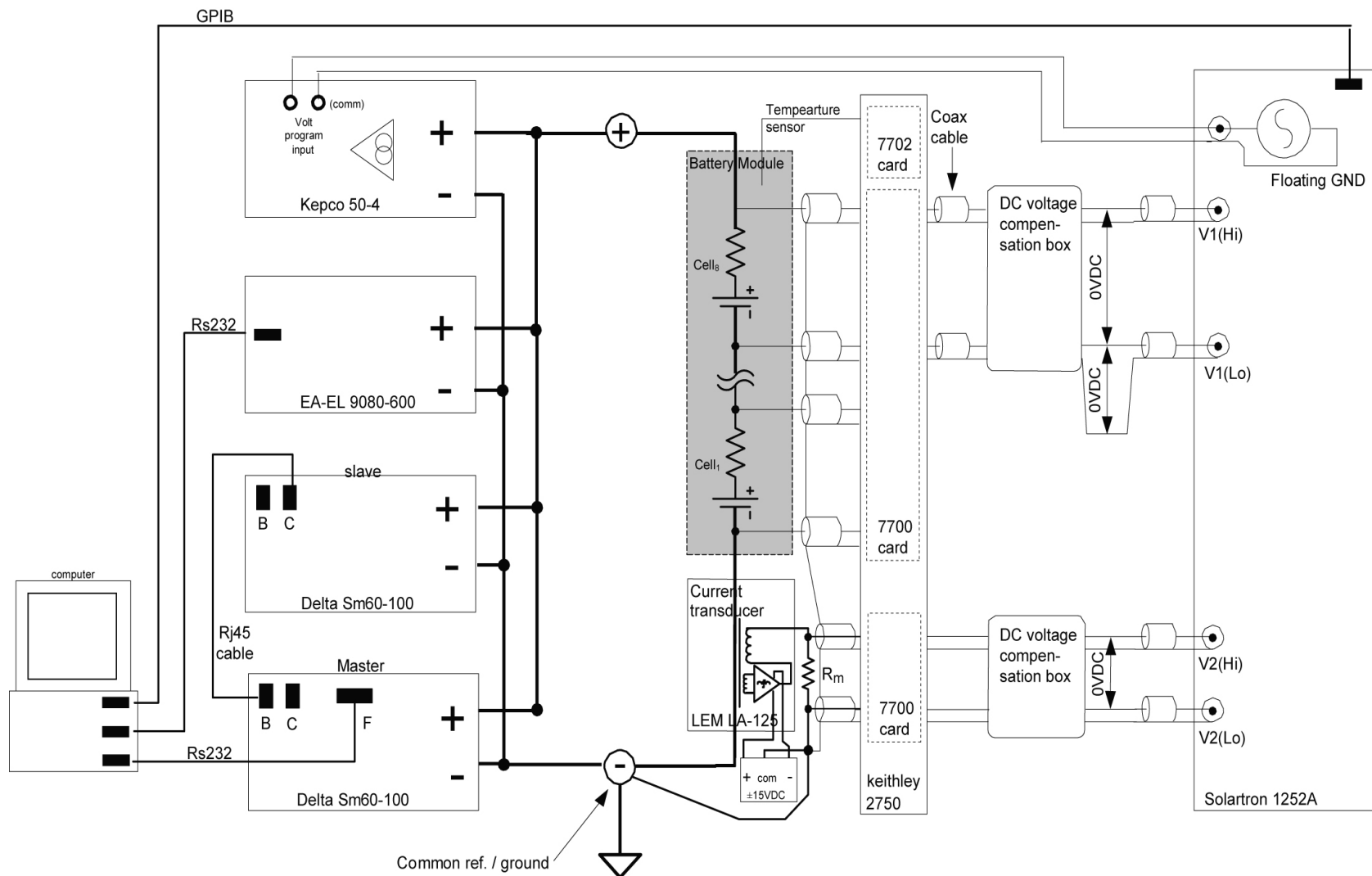
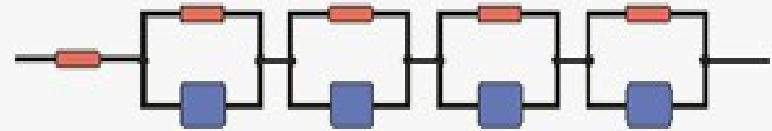


Laplace transform and pressurized SOC tests

Battery Module Test Setup





Impedance Spectroscopy

Ph.D. Course 45105 (2011)
MR 4. B779, Risø Campus
30/11/2011

**Module 11: Step response
and integral transforms**

Fourier Transform



- Fourier transform

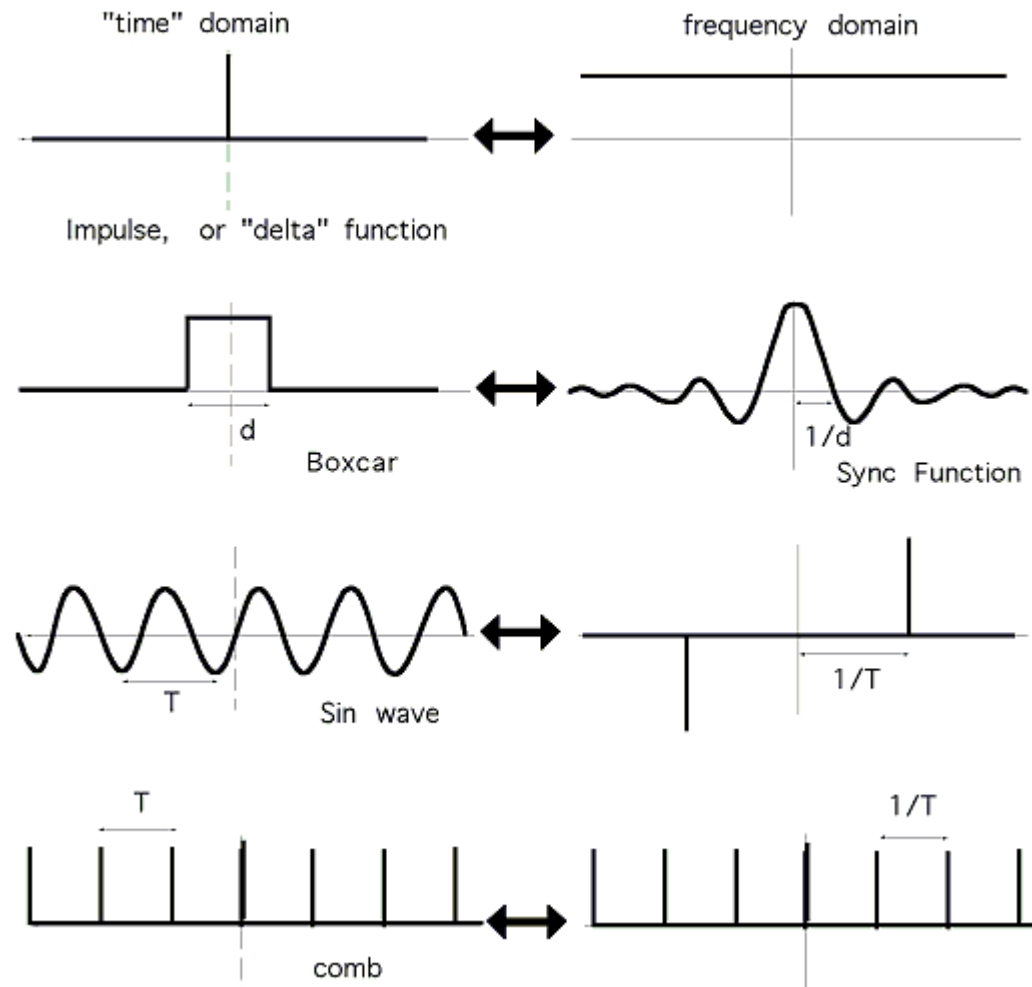
$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx,$$

$$\mathcal{F}(f(\mathbf{x})) = F(\omega) = \int f(\mathbf{x}) e^{-2\pi i \omega \mathbf{x}} d\mathbf{x}.$$

- Inverse Fourier transform

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi i x \xi} d\xi,$$

Fourier Transform



Laplace Transform



- *Two Different Impedance Measurement Techniques:*
 - First technique:
Apply a sinusoidal (AC) voltage (or current) and measure the resulting AC current (or voltage)

Strength: Very good noise rejection, if measured over several cycles

Weakness: Very long data acquisition time at low frequencies

- *Two Different Impedance Measurement Techniques:*
 - Second technique:
Apply a step voltage (or current) and measure the resulting current (or voltage) as a function of time

Strength: Very fast measurements at low frequency

Weakness: Difficult to get good precision at high frequencies. Less noise rejection

- What is the Laplace transform?

$$F(s) = \int_0^{\infty} e^{-s \cdot t} \cdot f(t) dt$$

- The transformation is essentially bijective for the majority of practical uses
- Common pairs of $f(t)$ and $F(s)$ can be found in look-up tables

- *Various functions and their laplace transform:*

Time domain	Laplace domain
$u(t)$	$\frac{1}{s}$
$e^{-\alpha t} \cdot u(t)$	$\frac{1}{s + \alpha}$
$(1 - e^{-\alpha t}) \cdot u(t)$	$\frac{\alpha}{s(s + \alpha)}$
$t \cdot u(t)$	$\frac{1}{s^2}$

$$s = \delta + \omega \cdot i$$

How to measure the impedance?

- Apply a step current $I(t) = I_0 \cdot u(t)$ to the cell
- Laplace transform $I(t)$ to obtain $I(s)$
- Measure $V(t)$
- Model $V(t)$ with a sum of exponential decay-functions
- Laplace transform the sum of exp. decay-functions to get $V(s)$
- $Z(s) = V(s)/I(s)$

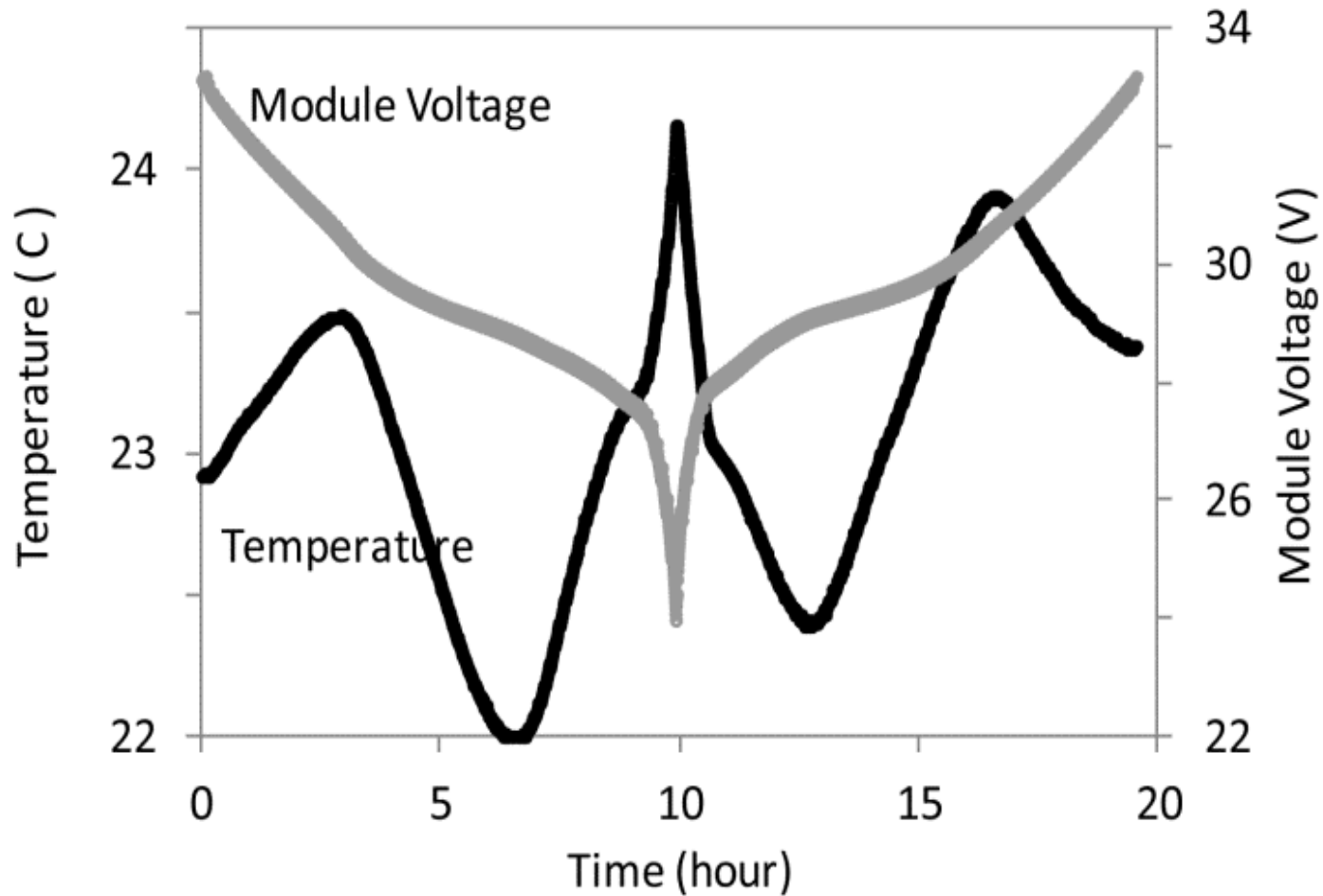
Laplace Transform



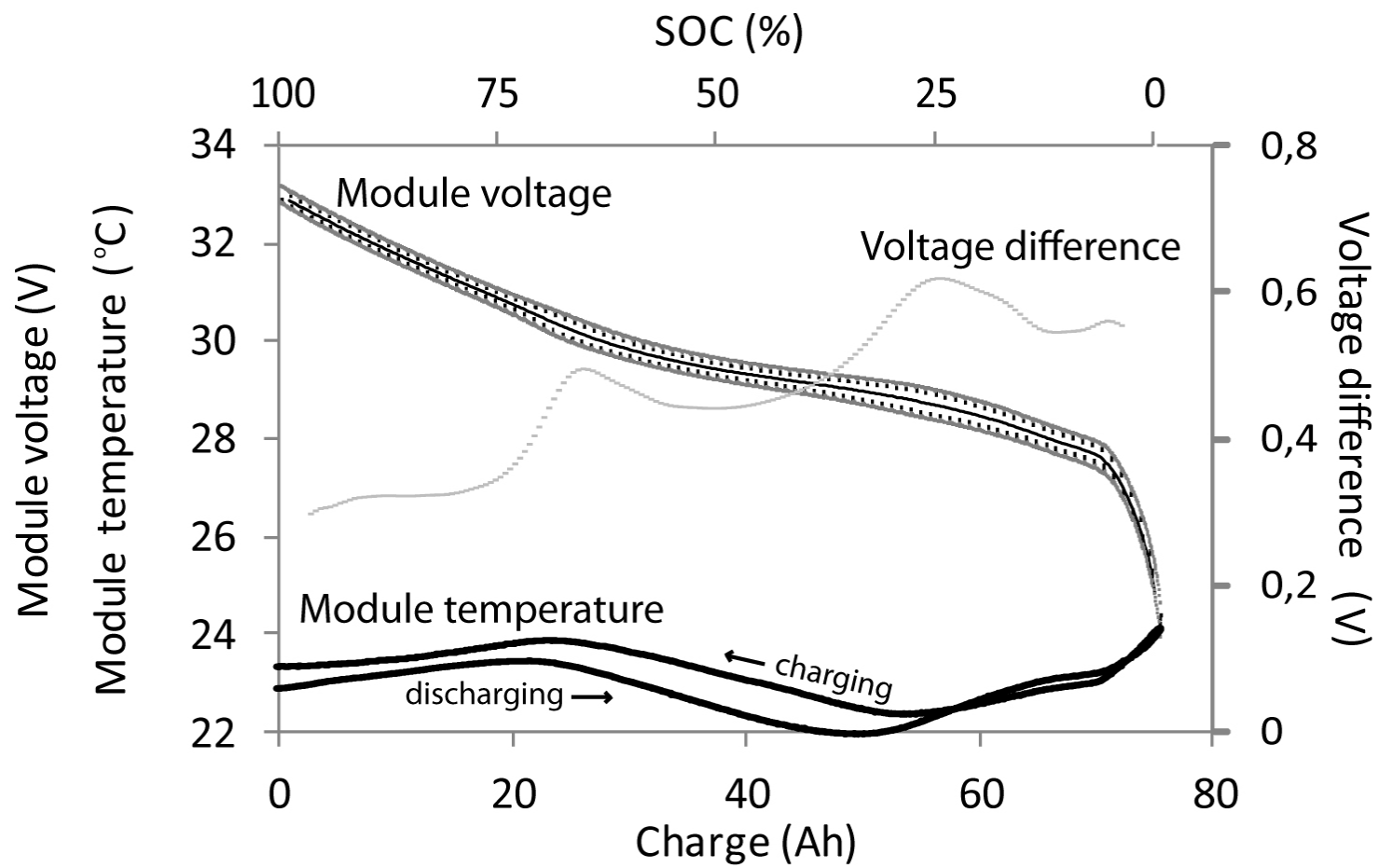
How to calculate $V(t)$ from $Z(s)$ and a current step function $I(t) = I_0 \cdot u(t)$?

- $I(s) = I_0/s$
- Multiply $Z(s)$ with $I(s)$ to get $V(s)$
- Model $V(s)$ with a sum of Laplace transformed exponential functions
- Inverse Laplace transform the sum of Laplace transformed exponentials to obtain $V(t)$

Charge Discharge Curve of KOKAM Battery Module: 75 Ah, 29,6V (8S) NMC



U, T vs SOC

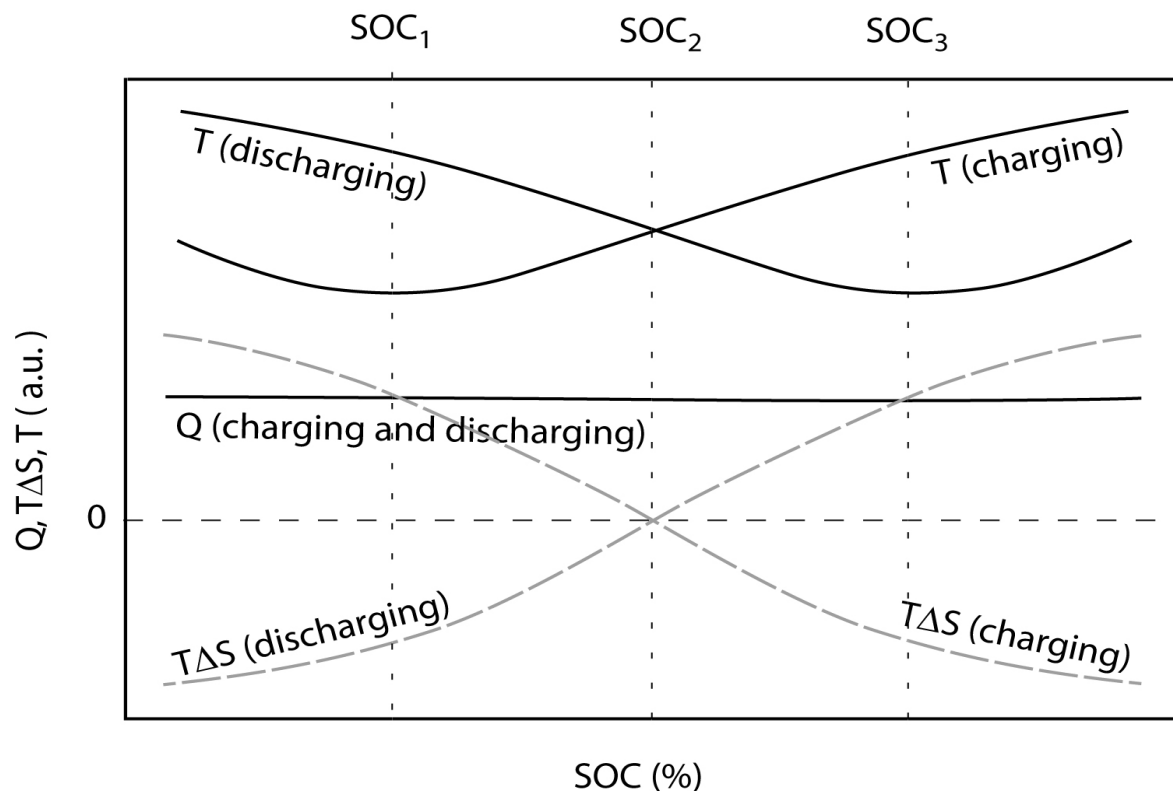


Endothermic/Exothermic Reaction Heat and Joule Heat

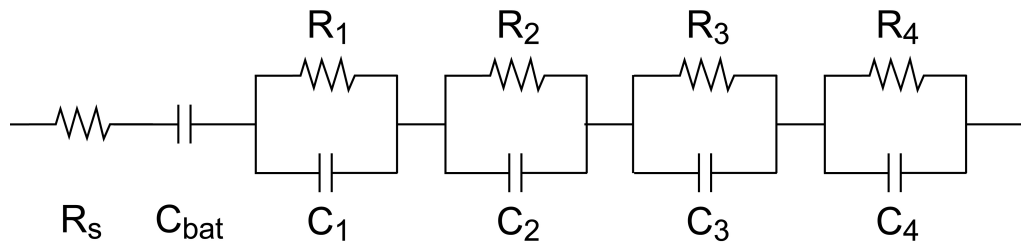
$$P_e = \frac{I}{nF} \cdot T \Delta S$$

$$P_J = R_i I^2$$

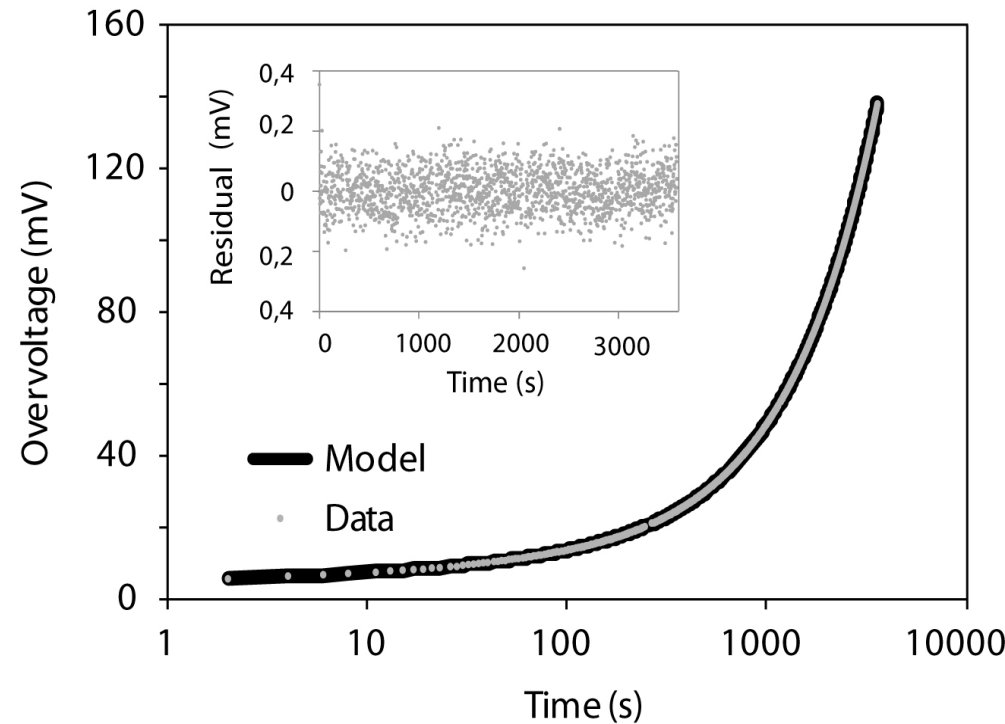
$$\frac{dT}{dt} = (P_e + P_J)/C_p$$



Overvoltage vs time after onset of 1A step-current



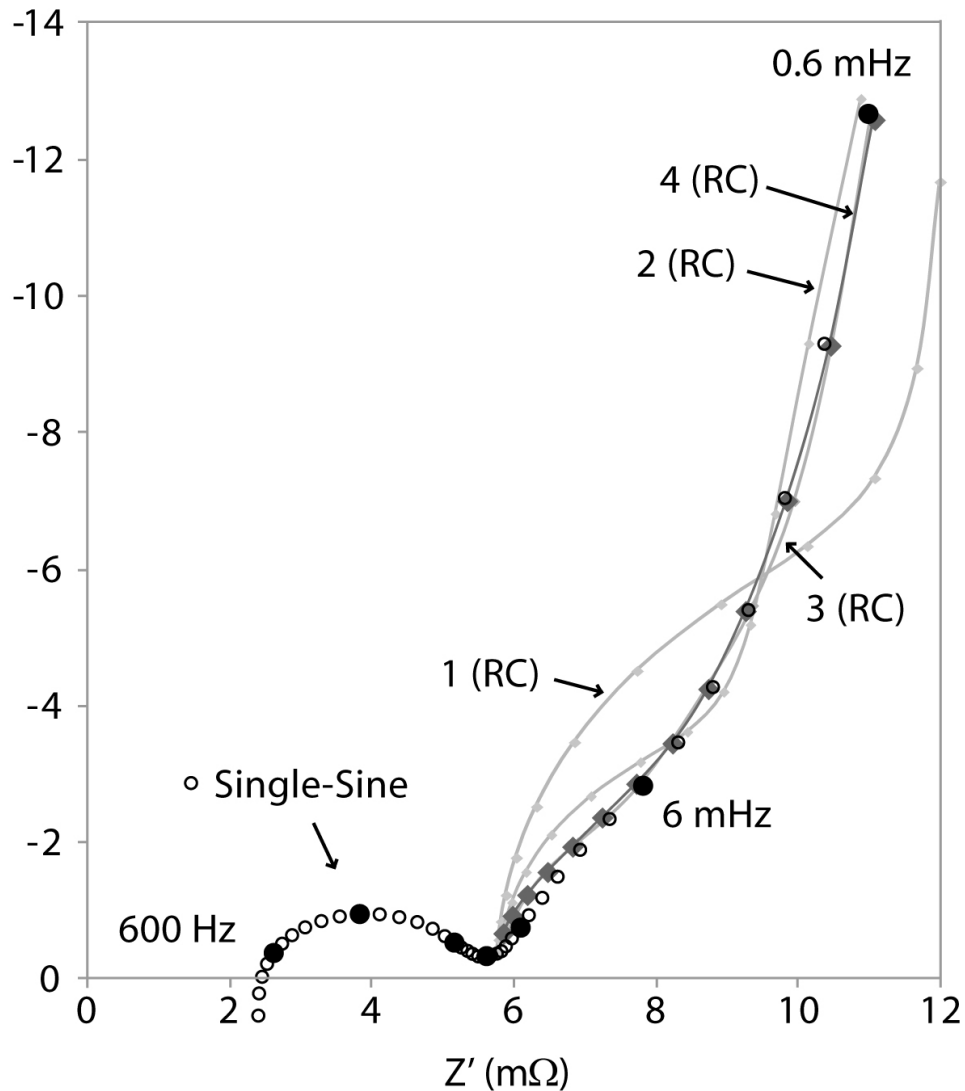
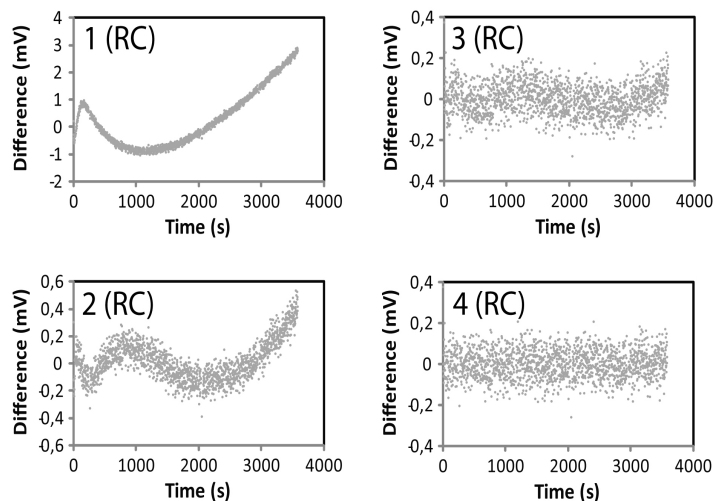
$$U_m(s) = I(s) \cdot Z_m(s) = \frac{I_0}{s} \cdot \left(R_s + \frac{1}{sC_{bat}} + \frac{R_1}{1 + sR_1C_1} + \frac{R_2}{1 + sR_2C_2} + \frac{R_3}{1 + sR_3C_3} + \frac{R_4}{1 + sR_4C_4} \right)$$



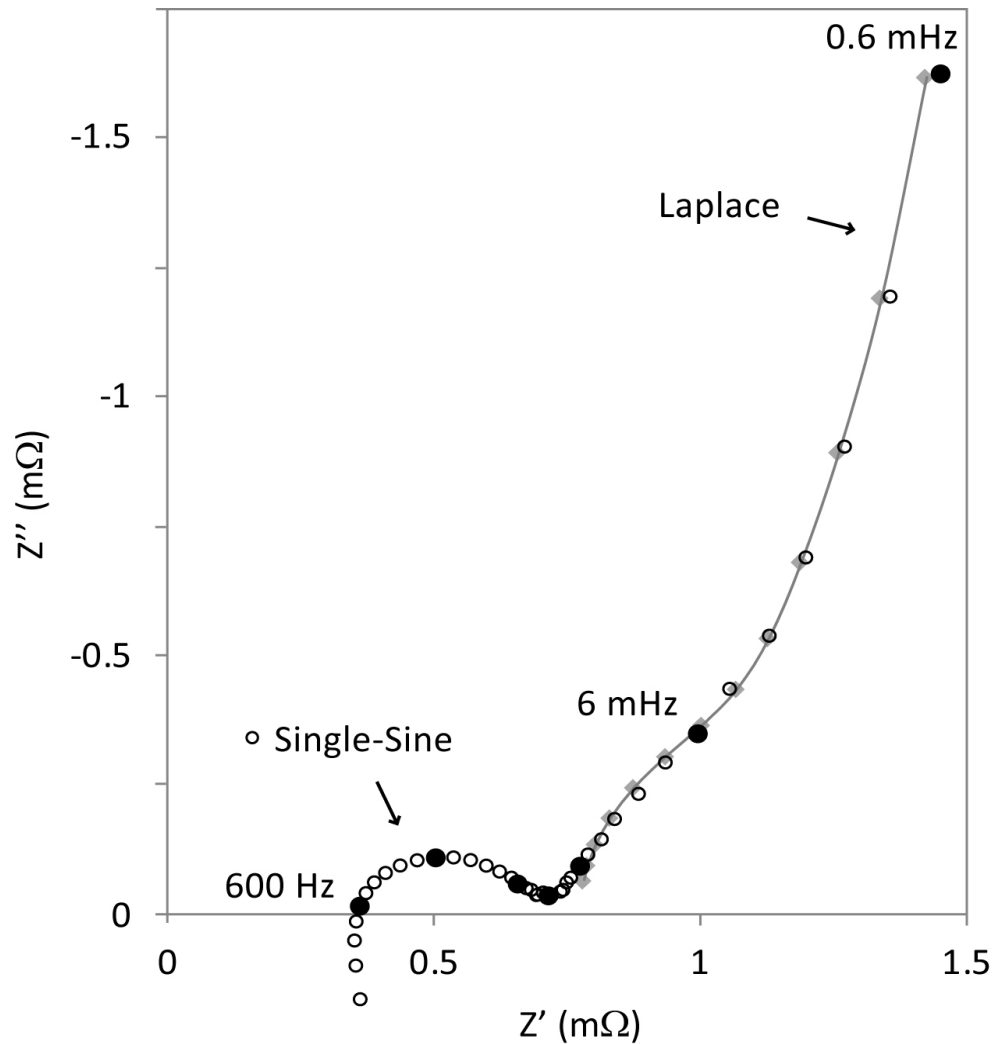
$$U_m(t) = u(t) \times I_0 \left[R_s + t \times C_{bat} + R_1 \left(1 - e^{-\frac{t}{R_1 C_1}} \right) + R_2 \left(1 - e^{-\frac{t}{R_2 C_2}} \right) + R_3 \left(1 - e^{-\frac{t}{R_3 C_3}} \right) + R_4 \left(1 - e^{-\frac{t}{R_4 C_4}} \right) \right]$$

Impedance spectra - Module

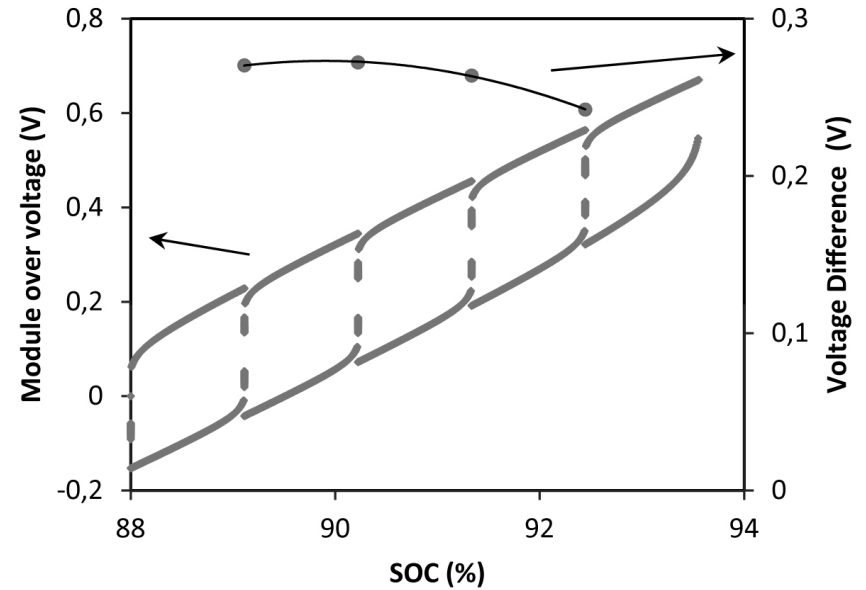
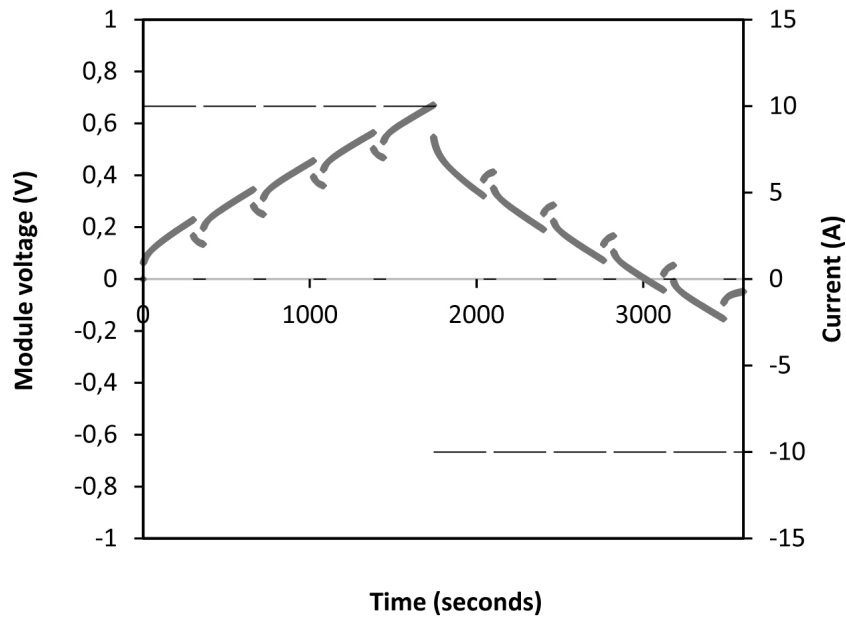
Residuals



Impedance spectra – Single Cell



Calculated Voltage Difference



Thank You

