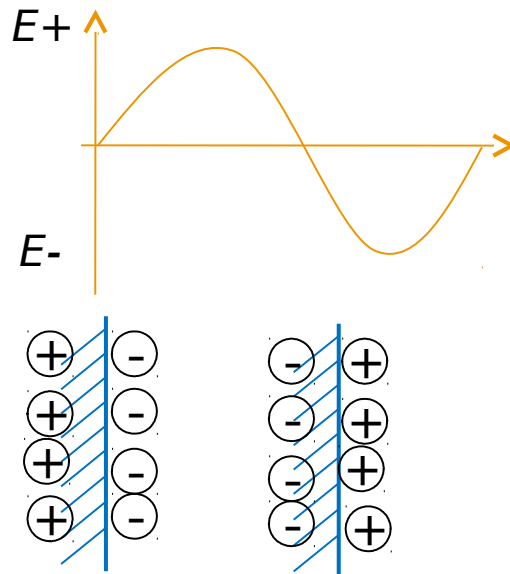
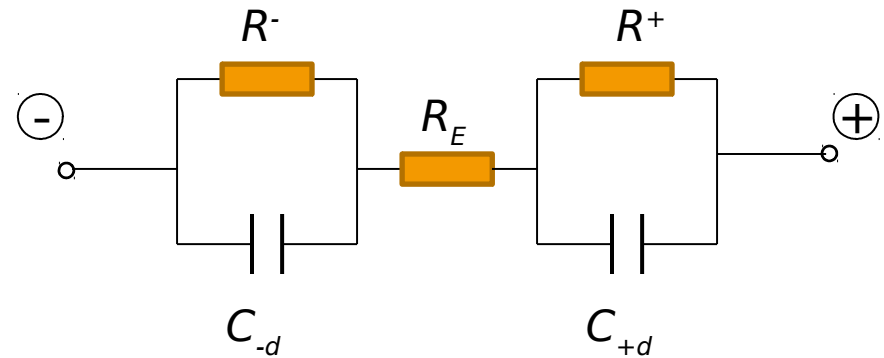


Interpretation of Electrochemical Impedance

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DTU Energy Conversion
Risø Campus, Roskilde, Denmark
www.energyconversion.dtu.dk



$$\Delta E = 0 \quad \Delta S \geq 0 \quad \int_a^b \epsilon \Theta + \Omega \int \delta e^{i\pi} = \{2.7182818284\} \quad \chi^2 \quad \Sigma \quad ! \quad \gg \quad \text{etc.}$$

Basic impedance elements

Resistor



$$Z = R$$

Capacitor



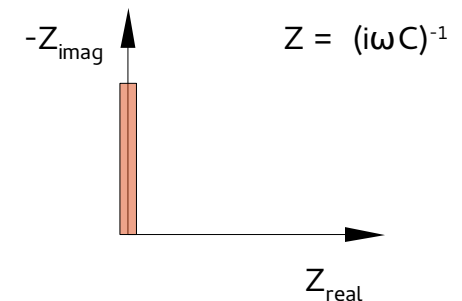
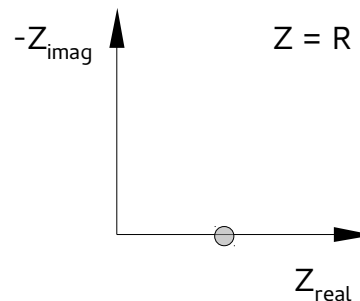
$$Z = \frac{1}{i\omega C} = \frac{-i}{\omega C}$$

Inductor



$$Z = i\omega L$$

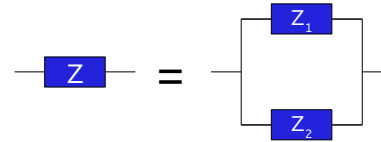
Angular frequency = $\omega = 2\pi f$



Parallel and Series Connections

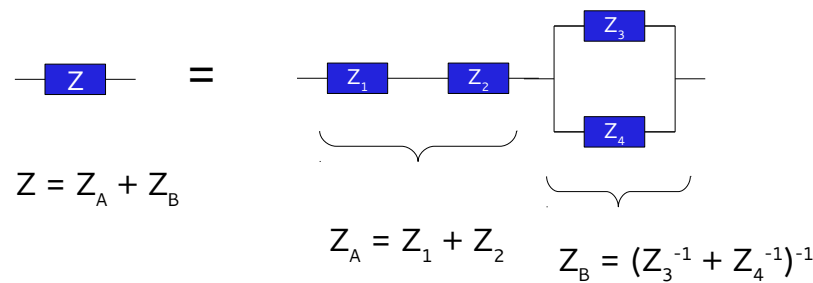
Parallel

$$1/Z = 1/Z_1 + 1/Z_2$$



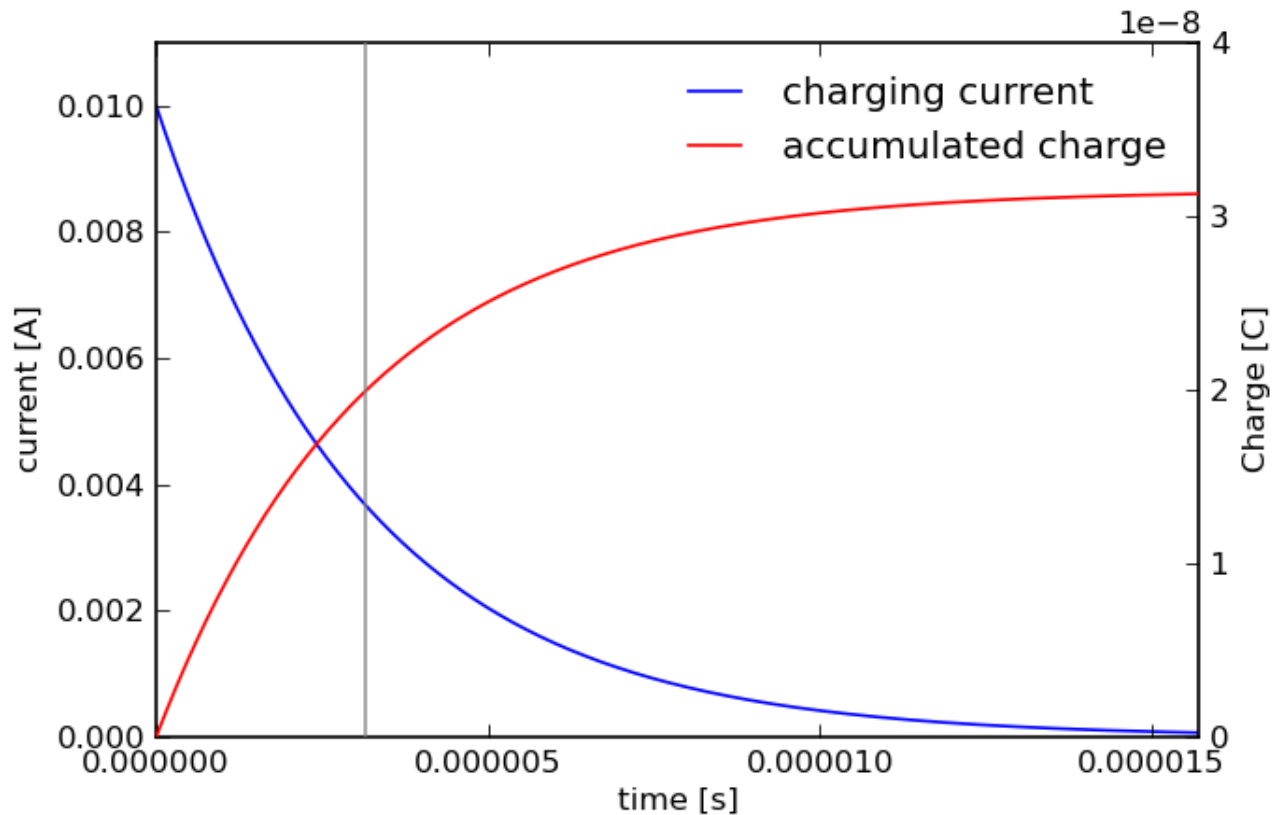
Series

$$Z = Z_1 + Z_2$$



Charging of a capacitor (time-domain)

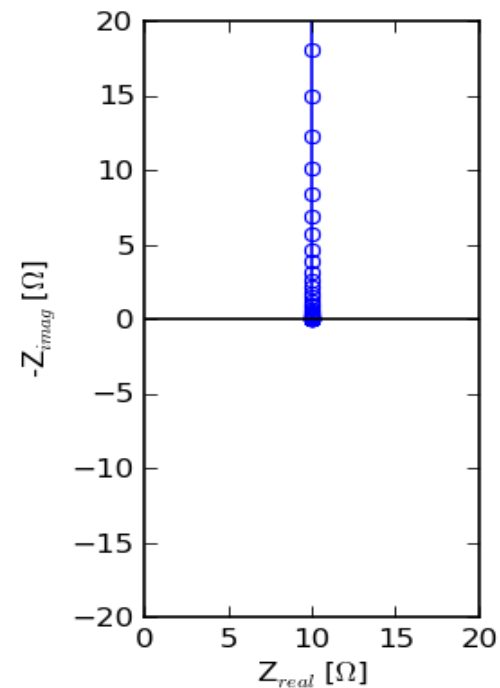
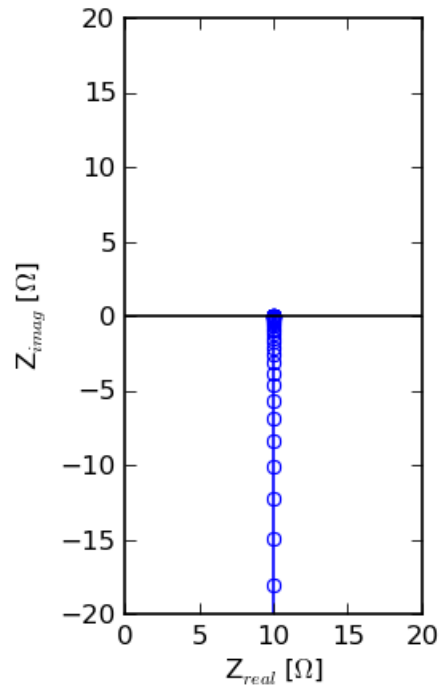
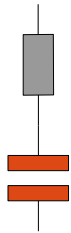
$$I_c = \frac{\Delta E}{R} e^{-t/RC}$$



Impedance -(R)-(C)-

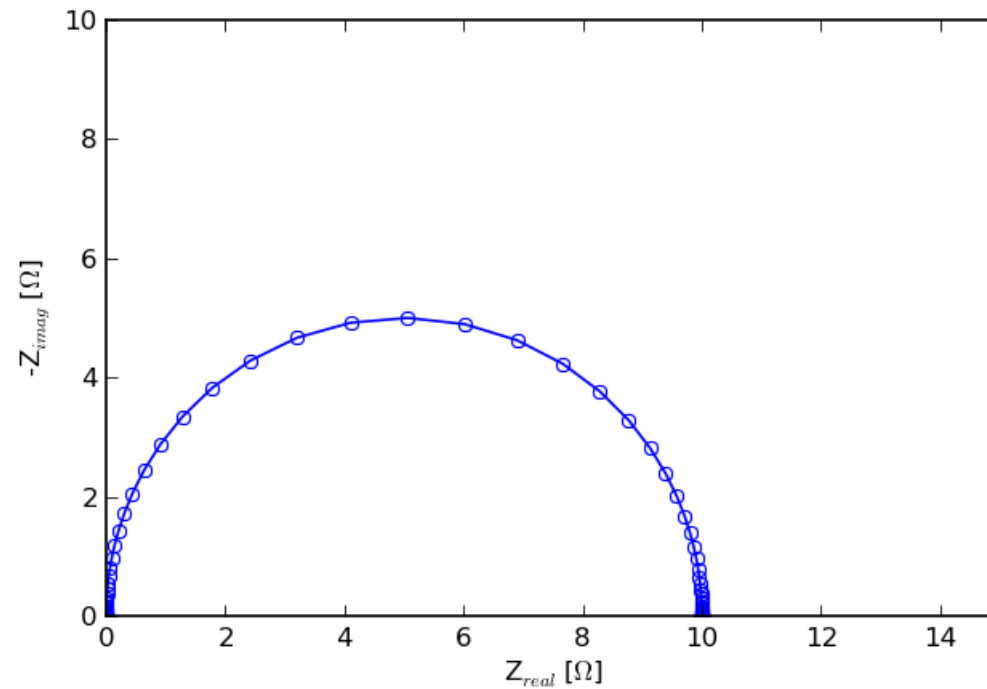
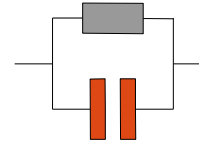
Series
connected
circuit

$$Z = R + \frac{-i}{\omega C}$$



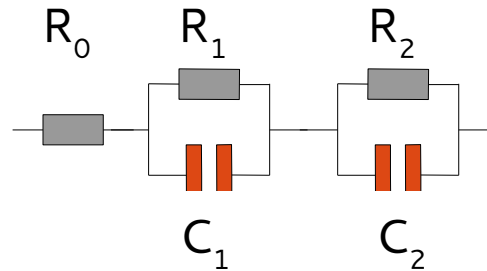
Impedance -(RC)-

Parallel
connected
circuits

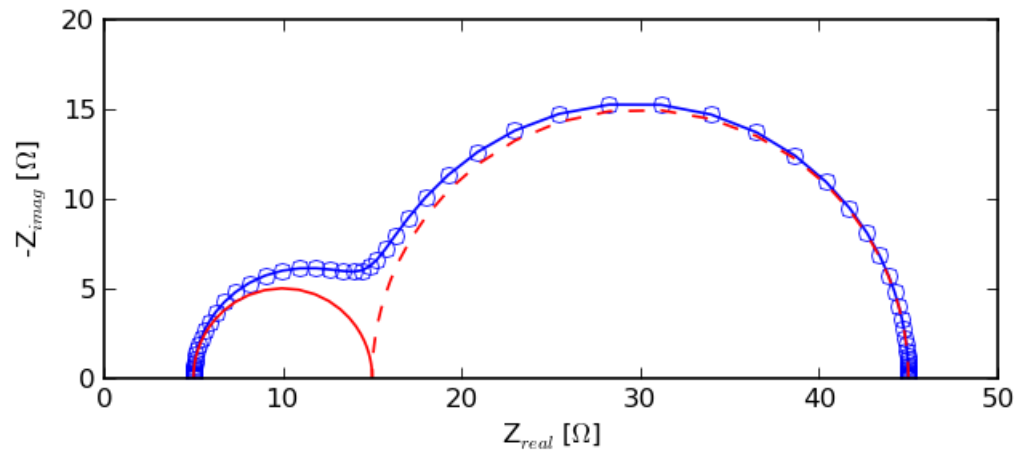


Impedance R_0 -(R_1C_1)-(R_2C_2)-

Parallel
connected
circuits



$$\begin{aligned} R_0 &= 5 \, \Omega \\ R_1 &= 10 \, \Omega \\ R_2 &= 30 \, \Omega \\ C_1 &= 1 \, \mu\text{F} \\ C_2 &= 10 \, \mu\text{F} \end{aligned}$$



"Nyquist plot"

"Complex plane plot"

Summit frequency $-(RC)-$

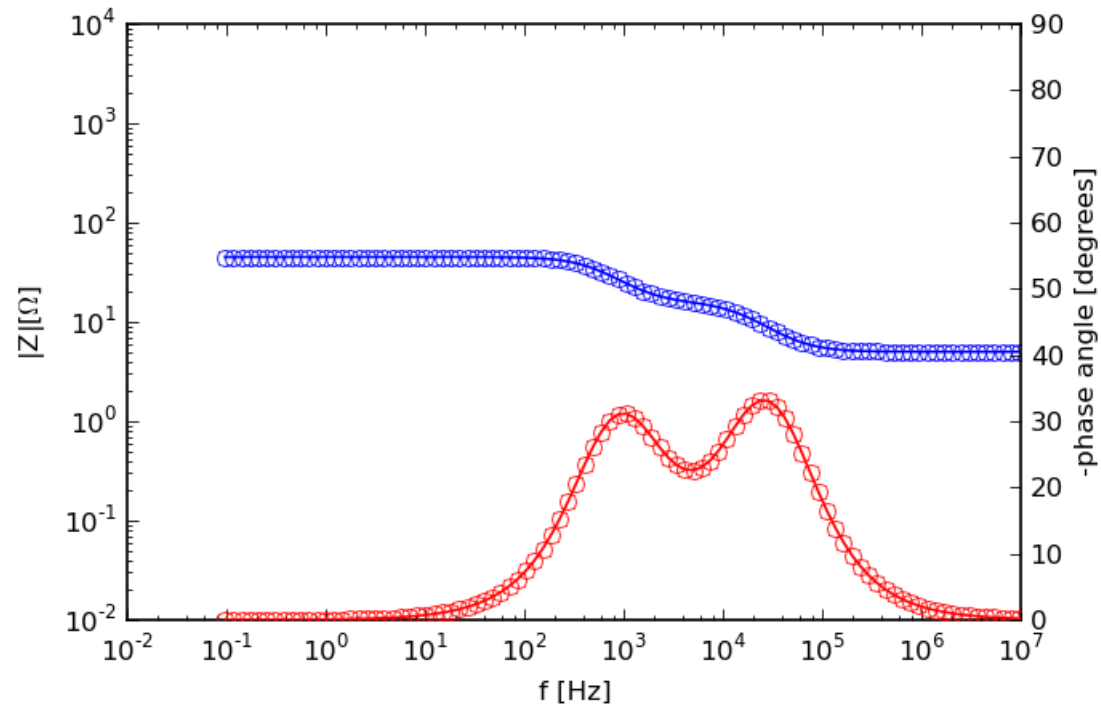
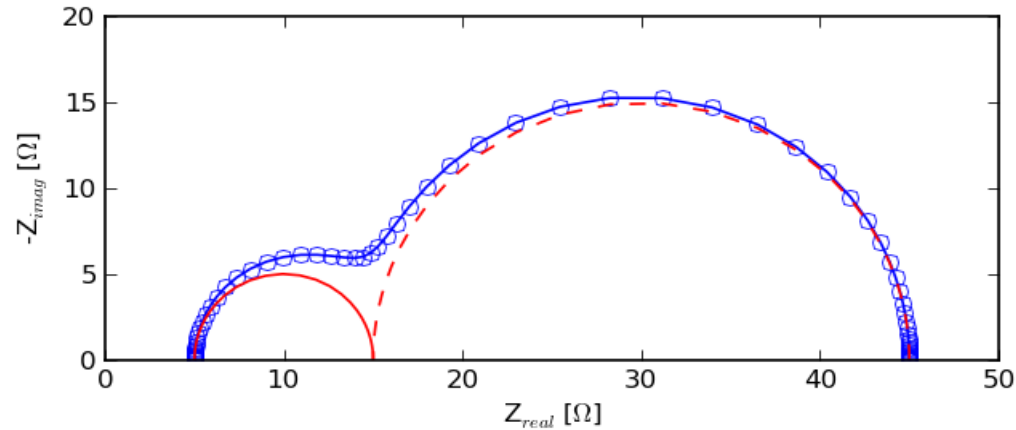
$$f_{\text{summit}} = 1/(RC)$$

Impedance R_0 -(R_1C_1)-(R_2C_2)-

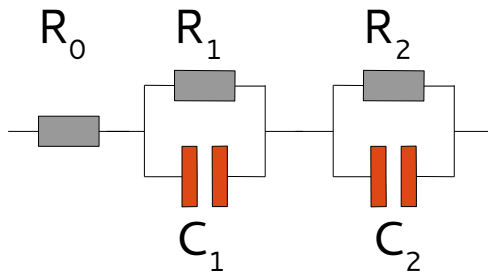
Parallel
connected
circuits

$$f_{\text{summit}} = 1/(2\pi RC)$$

$$\begin{aligned} R_0 &= 5 \, \Omega \\ R_1 &= 10 \, \Omega \\ R_2 &= 30 \, \Omega \\ C_1 &= 1 \, \mu\text{F} \\ C_2 &= 10 \, \mu\text{F} \end{aligned}$$



"Bode plot"

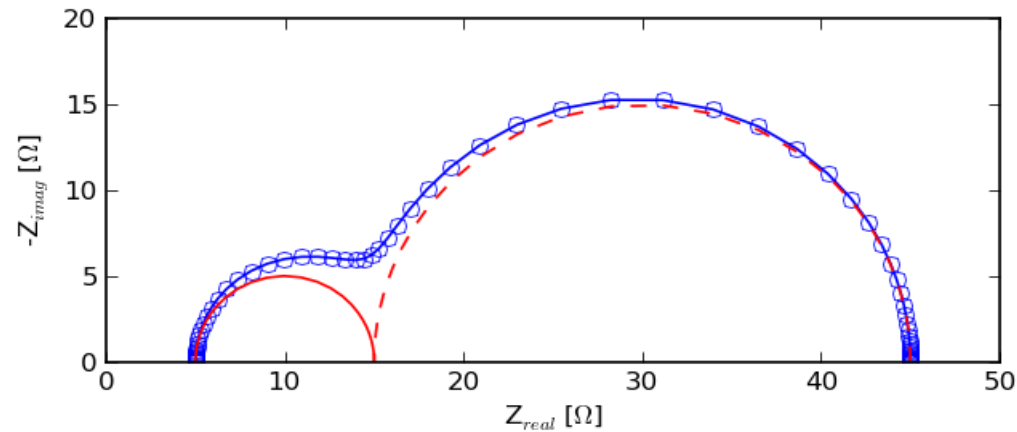
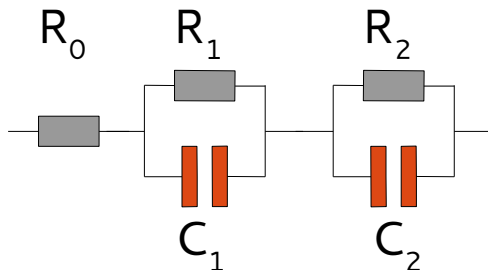


Impedance $R_0-(R_1C_1)-(R_2C_2)-$

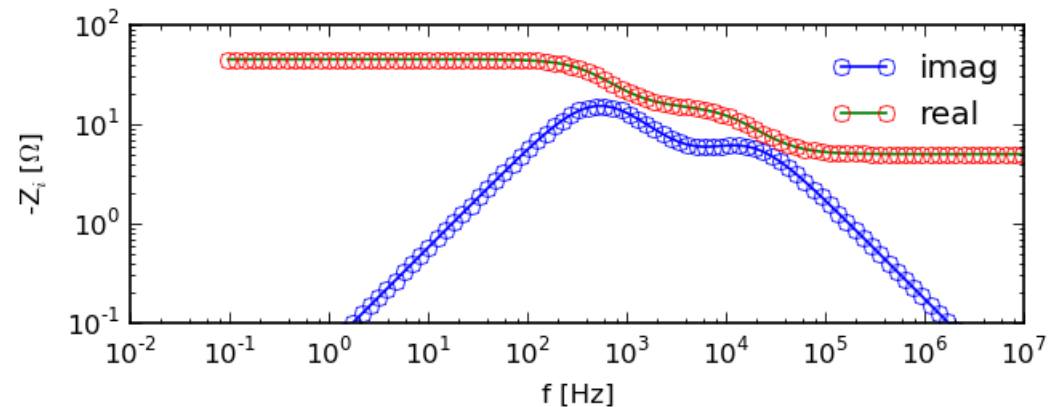
Parallel
connected
circuits

$$f_{\text{summit}} = 1/(2\pi RC)$$

"Bode plot"



$$\begin{aligned} R_0 &= 5 \, \Omega \\ R_1 &= 10 \, \Omega \\ R_2 &= 30 \, \Omega \\ C_1 &= 1 \, \mu\text{F} \\ C_2 &= 10 \, \mu\text{F} \end{aligned}$$



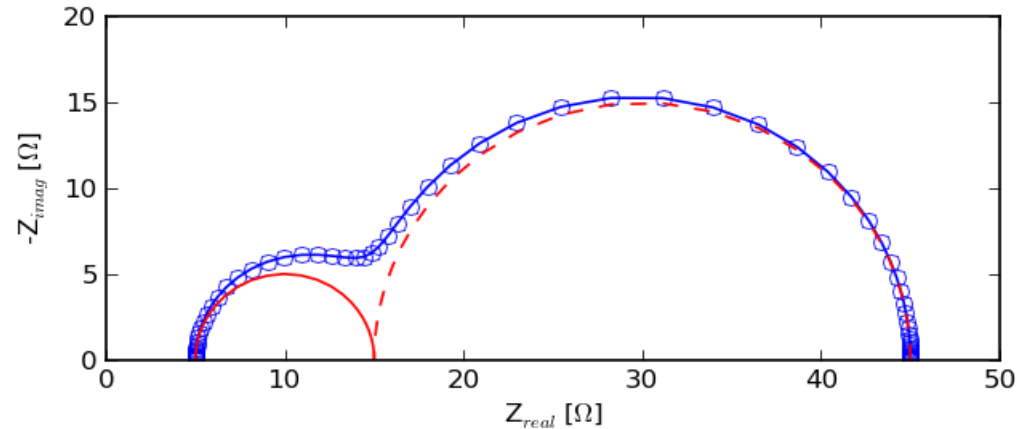
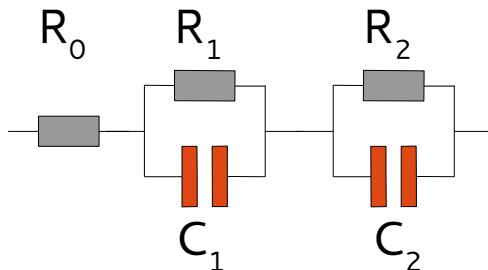
Impedance R_0 -(R_1C_1)-(R_2C_2)-

Parallel
connected
circuits

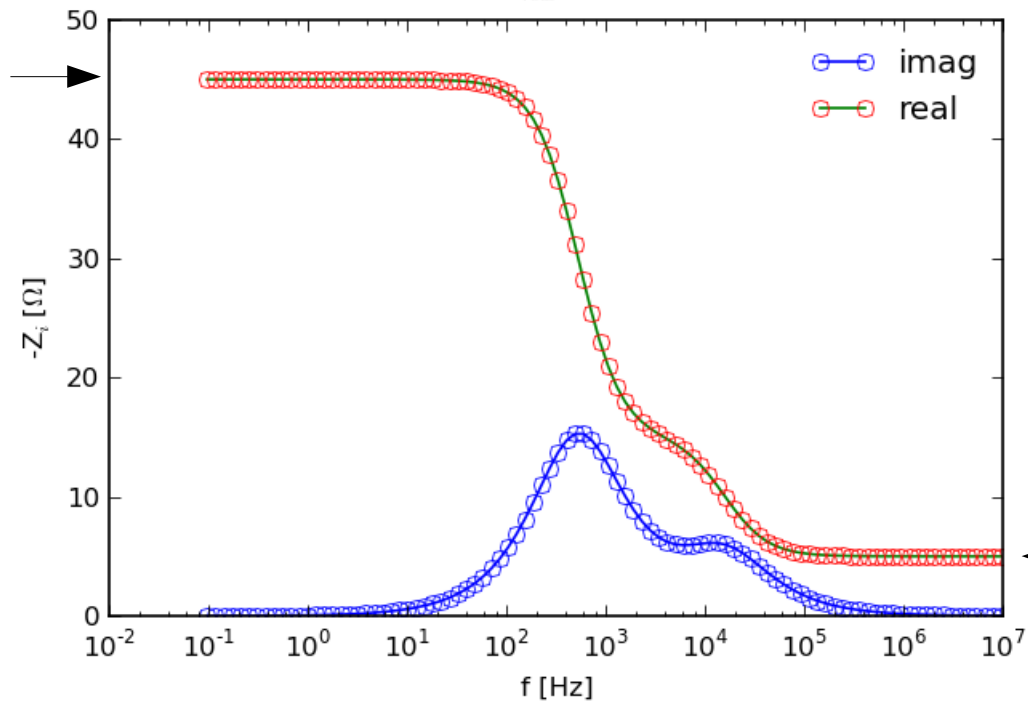
$$f_{\text{summit}} = 1/(RC)$$

$$R_0 + R_1 + R_2 \rightarrow$$

"Bode plot"



$$\begin{aligned} R_0 &= 5 \, \Omega \\ R_1 &= 10 \, \Omega \\ R_2 &= 30 \, \Omega \\ C_1 &= 1 \, \mu\text{F} \\ C_2 &= 10 \, \mu\text{F} \end{aligned}$$



R_0

Impedance – Constant Phase Element

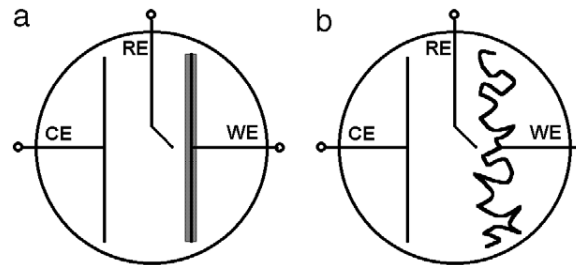
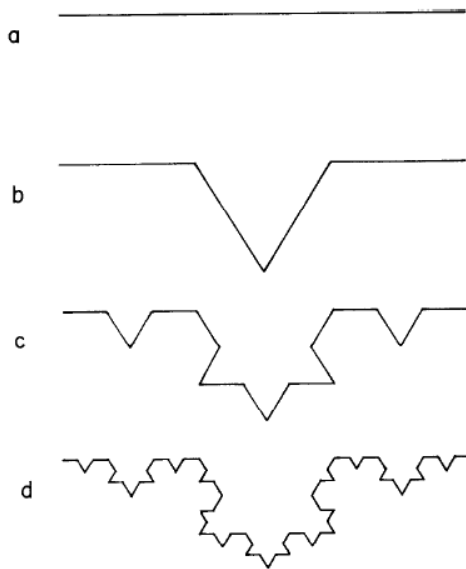
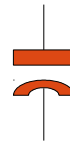
Capacitor



$$Z = \frac{1}{i\omega C} = \frac{-i}{\omega C}$$

Constant phase element
(most often denoted **Q** or CPE)

$$Z = \frac{1}{Q_0(i\omega)^\alpha}$$



T. Pajkossy, Solid State Ionics, 176, 1997–2003 (2005).

2D distributions, e.g.
-surface heterogeneities:
Crystal faces, variations in
surface properties...

3D distributions, e.g.
Geometry induced
non-uniform Potential and
Current distributions

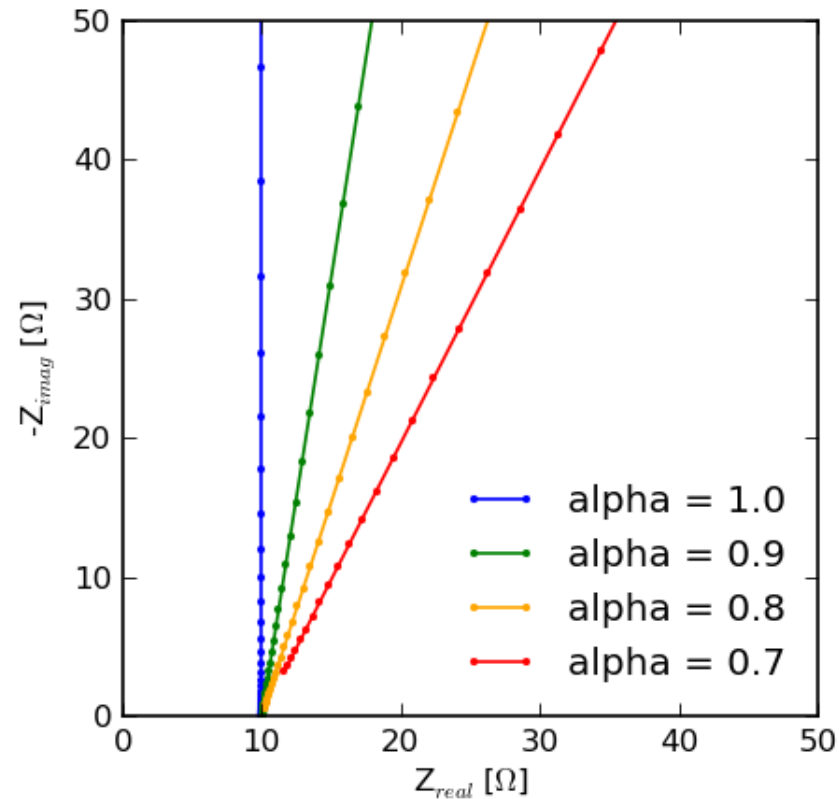
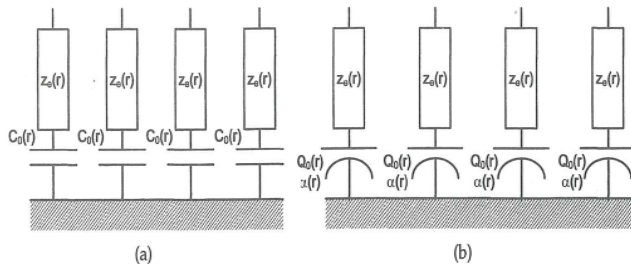
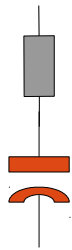
Distributions of activation
energies for transport or
Reaction, even a simple
resistance distribution
through a layer

R. de Levie, Journal of Electroanalytical Chemistry
and Interfacial Electrochemistry, 261, 1–9 (1989).

Constant Phase Element Properties

Distributed properties
leading to a frequency
(or time-constant)
dispersion

$$Z = \frac{1}{Q_0(i\omega)^\alpha}$$



Schematic from: M. Orazem and B. Tribollet; Impedance Spectroscopy

Constant Phase Element Properties

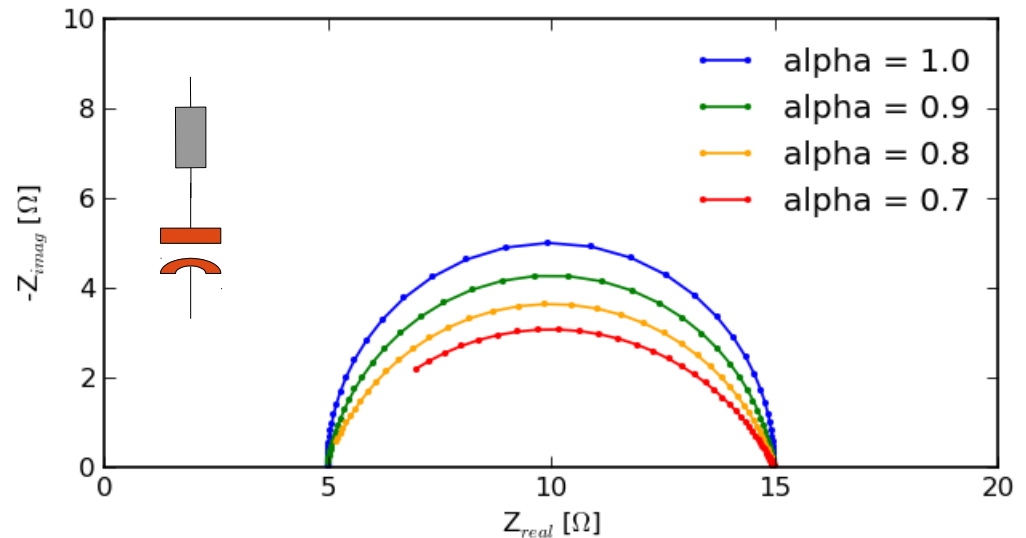
Distributed properties
leading to a frequency (or
time-constant) dispersion

"Equivalent" Capacitance $-(RQ)-$

$$C_{eq} = (RQ)^{1/n}/R$$

Summit frequency $-(RQ)-$

$$f_{summit} = 1/(2\pi(RQ_0)^{1/n})$$

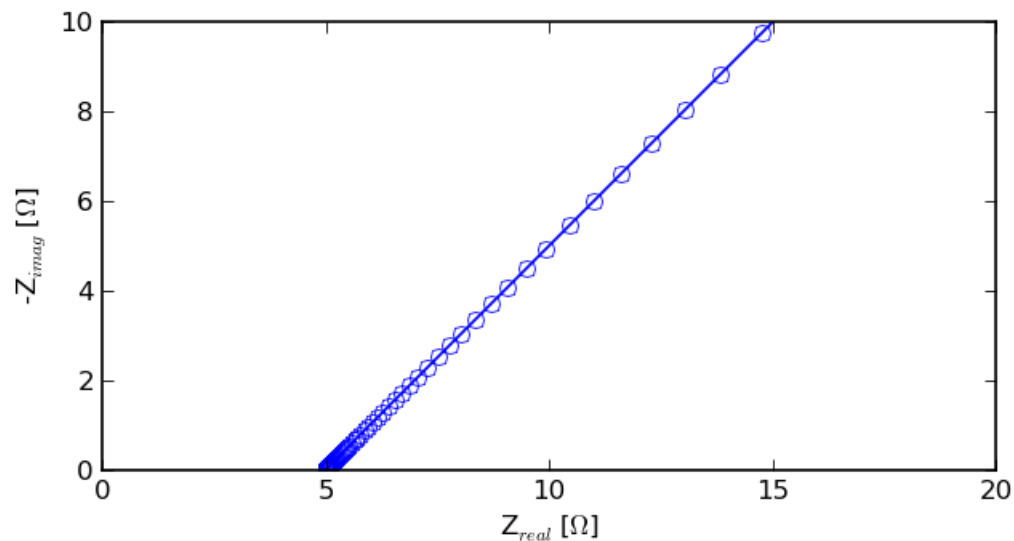
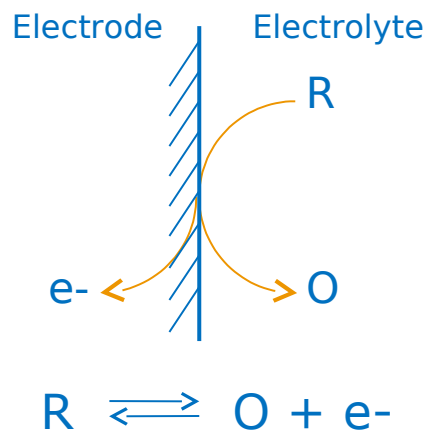


Semi-Infinite Warburg Diffusion Impedance

For a one-step, one-electron reaction $O + e^- \leftrightarrow R$

$$Z_{inf} = \sigma \omega^{-1/2} - i \sigma \omega^{-1/2}$$

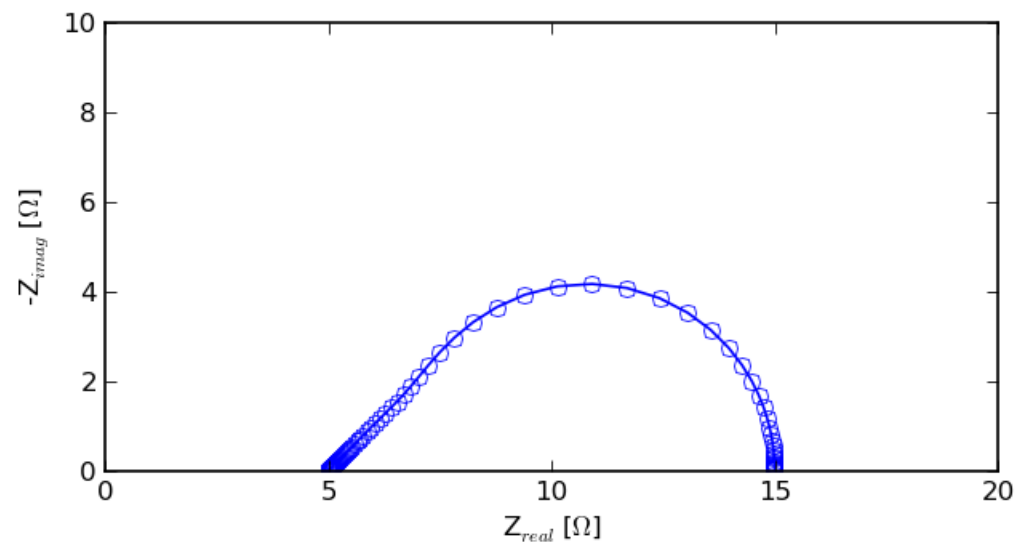
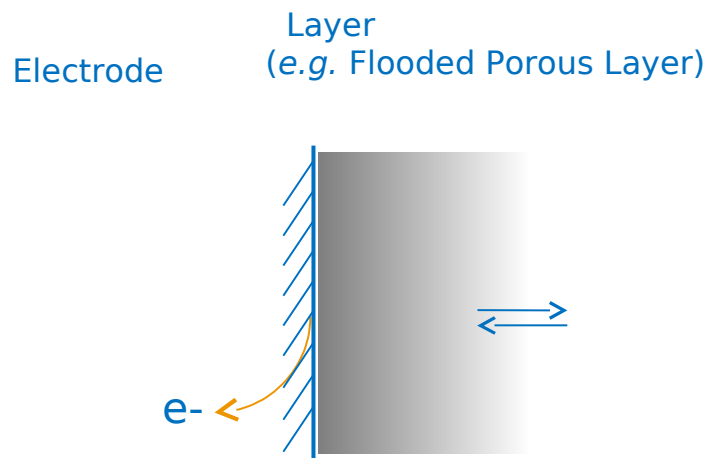
$$\sigma = \frac{RT}{F^2 A \sqrt{2}} \left(\frac{1}{D_O^{1/2} C_O^*} + \frac{1}{D_R^{1/2} C_R^*} \right)$$



Diffusion Impedance

Finite-length Warburg Diffusion Impedance

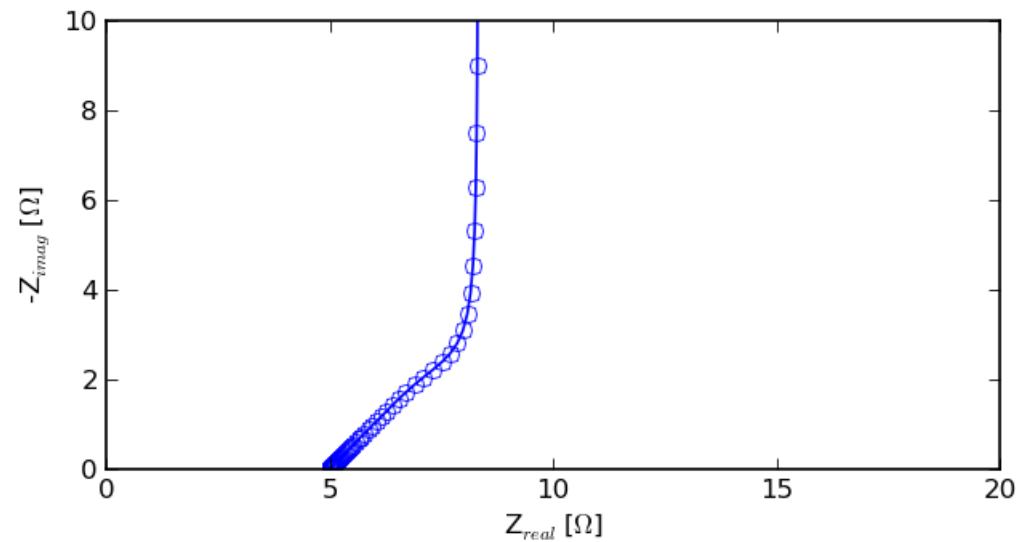
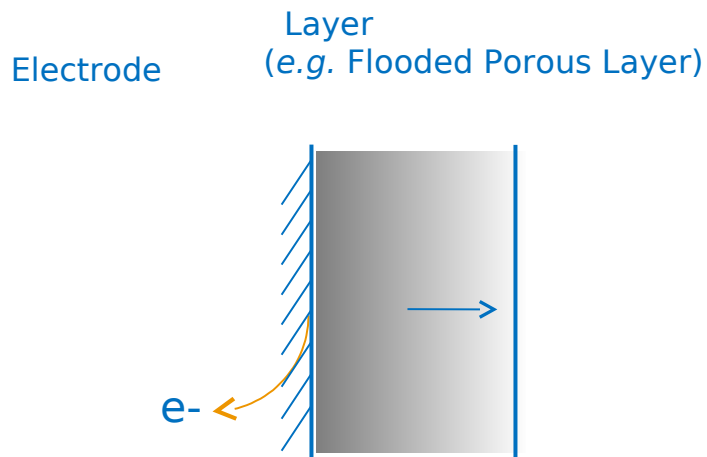
“non-blocking” outer interface



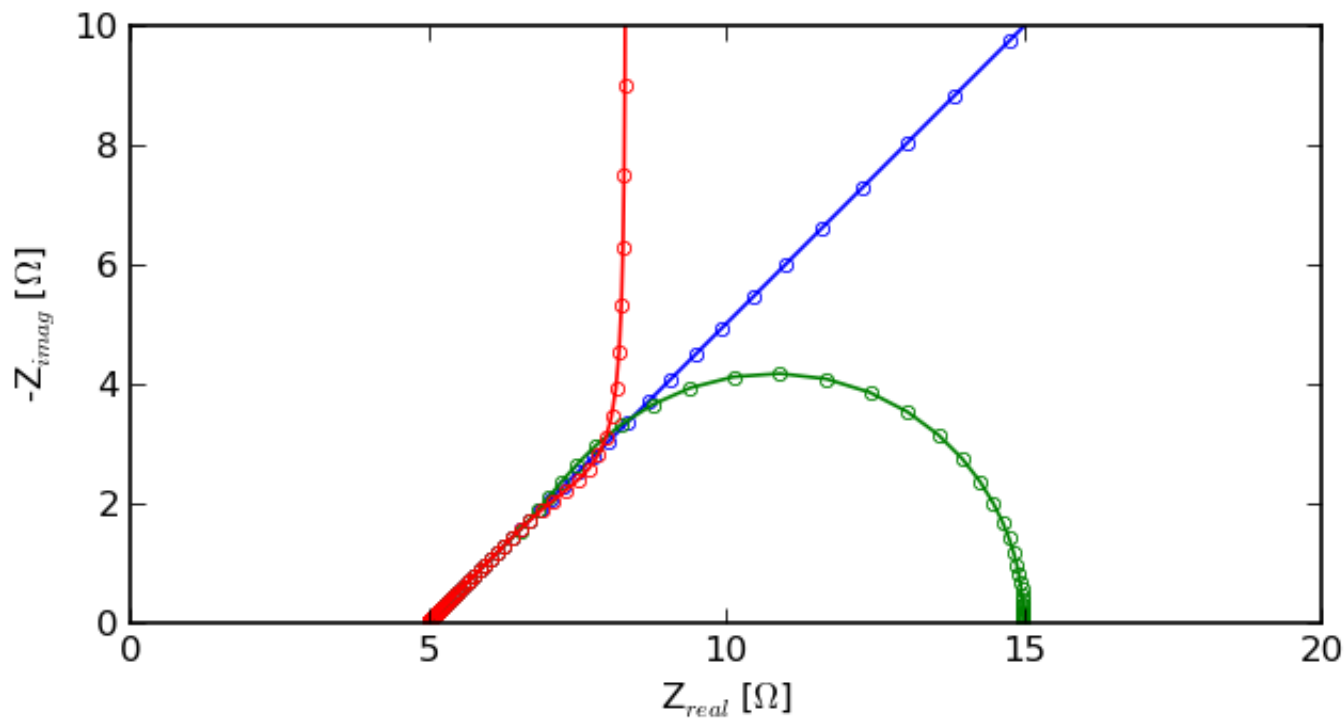
Diffusion Impedance

Finite-space Warburg Diffusion Impedance

“blocking” outer interface



Warburg Diffusion Impedances



The Randles-Circuit

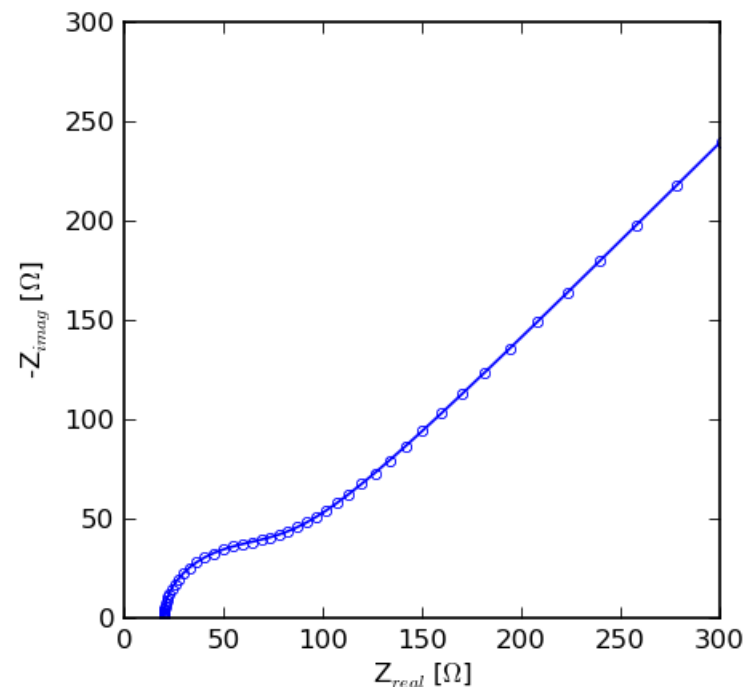
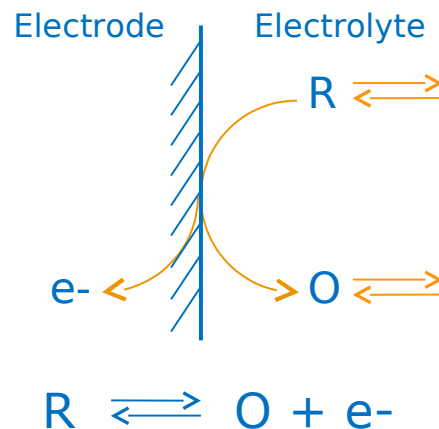
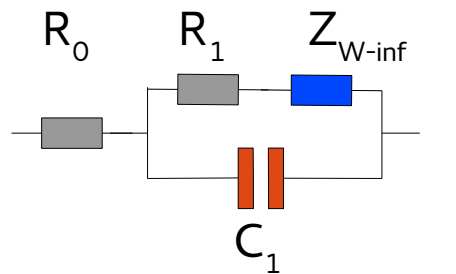
Semi-Infinite Warburg Diffusion Impedance

Charge Transfer Resistance parallel to Interfacial Capacitance

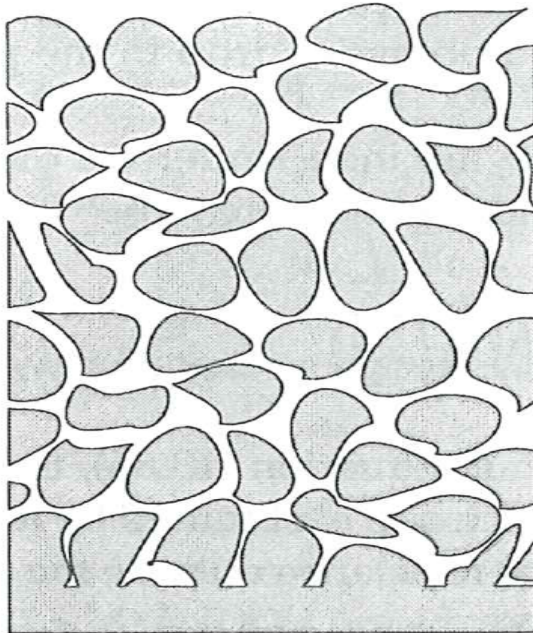
For a one-step, one-electron reaction $O + e^- \leftrightarrow R$

$$Z_{W-inf} = \sigma \omega^{-1/2} - i\sigma \omega^{-1/2}$$

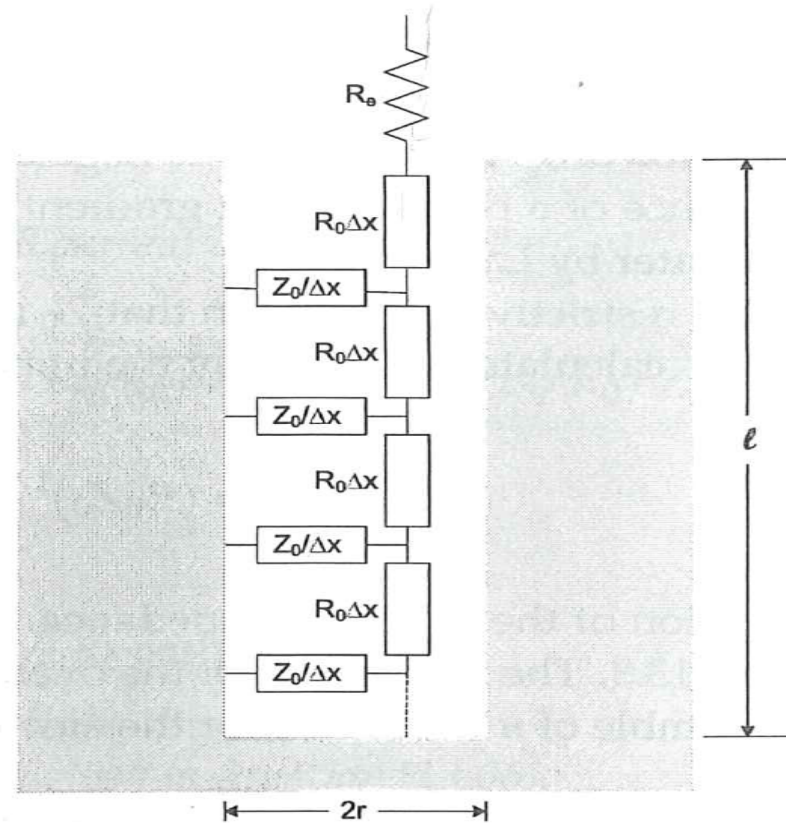
$$\sigma = \frac{RT}{F^2 A \sqrt{2}} \left(\frac{1}{D_O^{1/2} C_O^*} + \frac{1}{D_R^{1/2} C_R^*} \right)$$



Porous Electrodes (Transmission Lines)



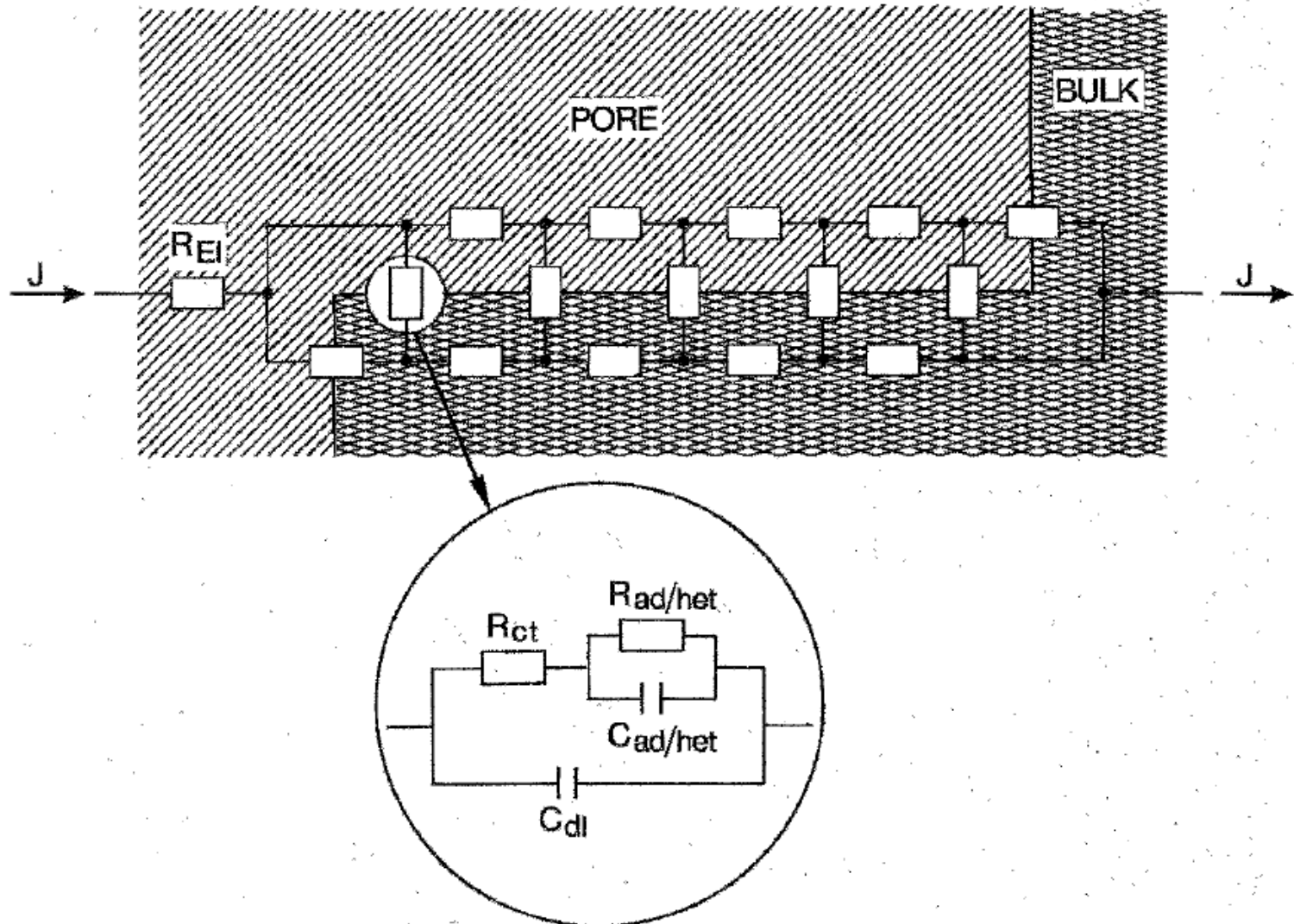
(a)



(b)

From: M. Orazem and B. Tribollet; Impedance Spectroscopy

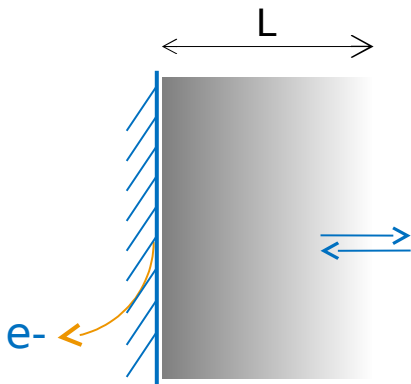
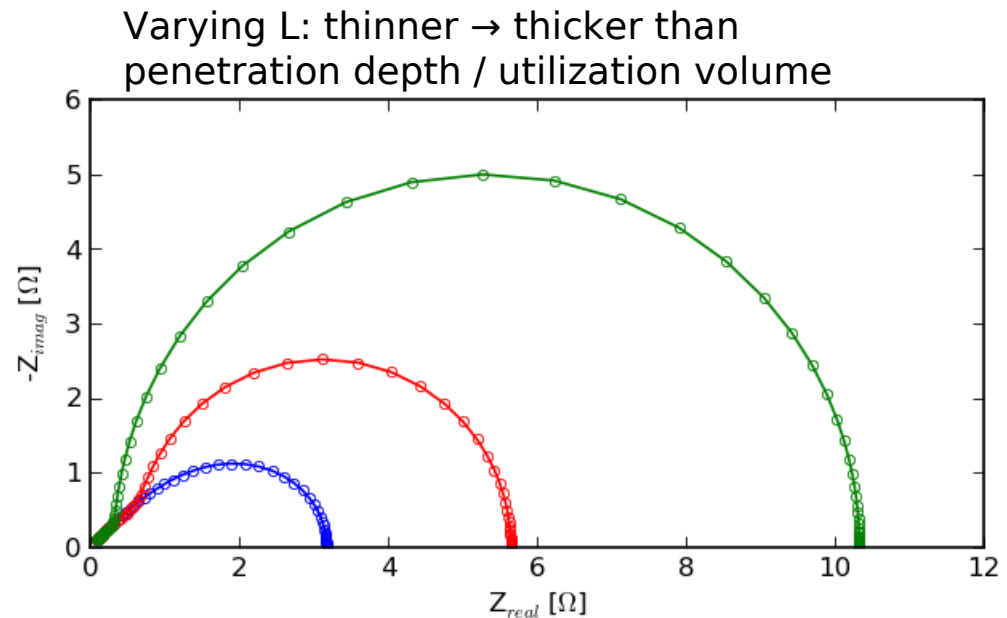
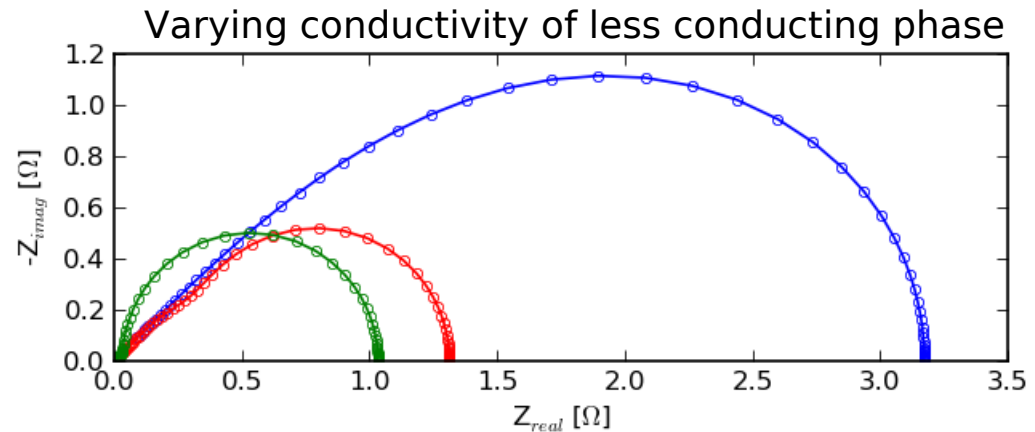
Porous Electrodes (Transmission Lines)



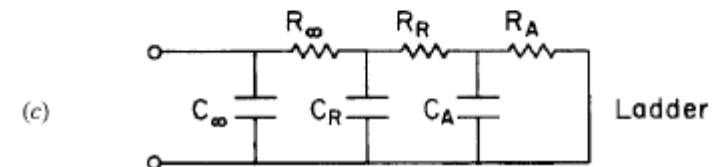
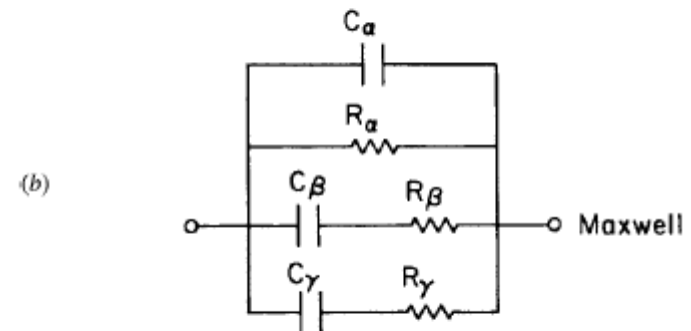
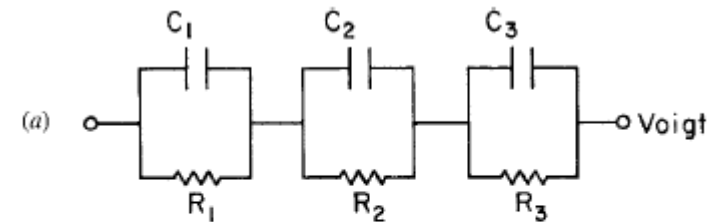
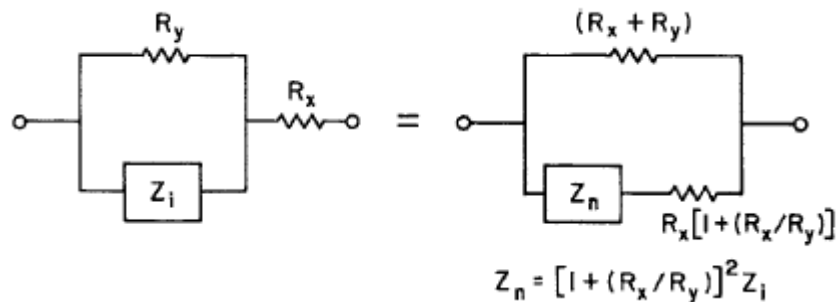
Source: Impedance Spectroscopy, 2nd Edition, Eds. E. Barsoukov & J. Ross Macdonald, John Wiley & Sons, Hoboken, NJ (2005), p.512

Porous Electrodes (Transmission Lines)

$$r_1 \gg r_2$$



Equivalence of Circuits



From: Impedance Spectroscopy, 2nd Edition, Eds. E. Barsoukov & J. Ross Macdonald, John Wiley & Sons, Hoboken, NJ (2005), p.94, 95.